Name:_____
M511: Linear Algebra (Spring 2018)
Instructor: Justin Ryan
Unit III Exam (Take Home): Chapter 6

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 8 points each.

1. Let $A, S \in \mathbb{R}^{n\times n}$ with $\det(S) \neq 0$, and define $B = S^{-1}AS$. Prove that A and B have the same eigenvalues. What is the relationship between the corresponding eigenvectors?

2. Let $A \in \mathbb{R}^{n\times n}$ be a diagonalizable matrix. Prove that

$$\det(A) = \lambda_1 \cdots \lambda_n.$$

Use this result to compute the determinant of the matrix.

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$$

3. Find the particular solution of the initial value problem.

$$\begin{cases} y_1' = y_1 - 2y_2, \\ y_2' = 2y_1 + y_2, \\ y_1(0) = 1, \;\; y_2(0) = -2 \end{cases}$$

4. Let A be a square matrix whose columns all add up to a fixed constant δ:

$$\sum_{i=1}^{n} a_{ij} = \delta \quad \text{for all } j = 1, \ldots, n.$$

Prove that δ is an eigenvalue of A.

5. Let $Q \in \mathbb{R}^{n \times n}$ be an orthogonal matrix: $Q^T Q = I$. Prove that *a.)* if λ is an eigenvalue of Q, then $|\lambda| = 1$; and *b.)* $\det(Q) = 1$.