Name:_____
M511: Linear Algebra (Spring 2018)
Instructor: Justin Ryan
Good Problems 2: Sections 3.2, 3.3

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Determine the null space of the matrix.

$$\begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{pmatrix}$$

2. Determine whether the following are subspaces of $\mathbb{P}_4$. Clearly explain your answers.

1. The set of polynomials in $\mathbb{P}_4$ of even degree.
2. The set of polynomials of degree 3.
3. The set of all polynomials $p \in \mathbb{P}_4$ such that $p(0) = 0$.
4. The set of all polynomials in $\mathbb{P}_4$ having at least one real root.

3. Which of the following are spanning sets for $\mathbb{P}_3$? Justify your answers.

1. $\{1, x^2, x^2 - 2\}$
2. $\{2, x^2, x, 2x + 3\}$
3. $\{x + 2, x + 1, x^2 - 1\}$
4. $\{x + 2, x^2 - 1\}$

4. Prove that any finite set of vectors that contains the zero vector must be linearly dependent.

5. Prove that any nonempty subset of a linearly independent set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is also linearly independent.