Name:__________________________________
M511: Linear Algebra (Spring 2018)
Instructor: Justin Ryan
Good Problems 3: Sections 3.4-3.5

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Let S be the subspace of $\mathbb{P}_3$ consisting of all polynomials p such that $p(0) = 0$, and let T be the subspace of all polynomials q such that $q(1) = 0$. Find bases for S, T, and $S \cap T$.

2. Show that if U and V are subspaces of $\mathbb{R}^n$ and $U \cap V = \{\mathbf{0}\}$, then

$$\dim(U + V) = \dim U + \dim V.$$

3. Given

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \quad S = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix},$$

find vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ so that S will be the transition matrix from V to U.

4. Write the standard coordinates in $\mathbb{P}_7$ for the 6^{th} degree Taylor polynomial of $f(x) = \cos(x)$.

5. Let $U = \{x, 1\}$ and $V = \{2x-1, 2x+1\}$ be ordered bases for $\mathbb{P}_2$. Find the transition matrices $U \to V$ and $V \to U$.