Name:______________________________

M511: Linear Algebra (Spring 2018)

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Good Problems 4: Sections 2.1, 3.5, and 3.6

WICHITA STATE UNIVERSITY

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ 2 & 3 & 2 \end{pmatrix}.$$

a.) Find all minors of A.

b.) Find all cofactors of A.

c.) Compute the determinant of A.

d.) Compute A^{-1}, if it exists. If it doesn't exist, explain why.

2. Consider the subspace of $C^\infty(\mathbb{R})$, $S = \text{span}\{e^x, e^{-x}\}$.

a.) Show that every function $\varphi \in S$ is a solution of the second order differential equation $y'' - y = 0$.

b.) Show that $V = \left\{\sinh(x), \cosh(x)\right\}$ also forms a basis for S.

c.) Regarding $E = \left\{e^x, e^{-x}\right\}$ as the standard basis for S, find the transition matrices from $V \to E$ and $E \to V$.

3. Consider the vectors in $\mathbb{R}^2$,

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \quad \text{and} \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

Letting $U = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $V = \{\mathbf{v}_1, \mathbf{v}_2\}$, find the transition matrices $U \to V$ and $V \to U$.

4. Consider the matrix

$$A = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix}.$$

Find bases for the row, column, and null spaces of A.