Name:________________________________

M511: Linear Algebra (Spring 2018)

Instructor: Justin Ryan

Good Problems 5:

WICHITA STATE UNIVERSITY

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

---

1. Consider the transformation $L : \mathbb{P}_3 \to \mathbb{P}_3$ defined by $L(p)(x) = x \cdot p'(x) + p(0)$.

a.) Prove that L is linear.

b.) Find the matrix representing L with respect to the ordered basis $\{1, (x-1), (x-1)^2\}$.

2. Let L be a linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^2$ satisfying

$$L\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \text{and} \quad L\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}.$$

a.) Write the matrices representing L with respect to the basis $U = \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$, and with respect to the standard basis of $\mathbb{R}^2$.

b.) Compute $L\begin{pmatrix} 4 \\ 4 \end{pmatrix}$, and write your answer in coordinates with respect to the basis $V = \left\{ \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \end{pmatrix} \right\}$.

3. Let $A \in \mathbb{R}^{3\times 5}$ with columns $\mathbf{a}_1, \ldots, \mathbf{a}_5$. Further suppose that $\mathbf{a}_1$ and $\mathbf{a}_3$ are linearly independent, $\mathbf{a}_2 = 2\mathbf{a}_1$, $\mathbf{a}_4 = \mathbf{a}_1 + \mathbf{a}_3$, and $\mathbf{a}_5 = \mathbf{a}_4 - \mathbf{a}_2$.

a.) What is the reduced row echelon form (RREF) of A?

b.) What is the column space of A?

4. Let $A = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ be a basis of $\mathbb{R}^4$ and $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis of $\mathbb{R}^3$. Suppose $L : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by

$$L(\mathbf{x}) = x_4\mathbf{b}_1 + x_2\mathbf{b}_2 + (x_1 - x_3)\mathbf{b}_3,$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4)^T$ is given in A-coordinates. Write the matrix representing L with respect to the bases A and B.