Name:_____

M511: Linear Algebra (Spring 2018)
Instructor: Justin Ryan
Good Problems 7: Section 5.1

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

---

1. Show that if $\mathbf{x}$ and $\mathbf{y}$ are any vectors in $\mathbb{R}^n$, then $\|\mathbf{x}+\mathbf{y}\|^2 \leq \big(\|\mathbf{x}\|+\|\mathbf{y}\|\big)^2$, and hence

$$\|\mathbf{x}+\mathbf{y}\| \leq \|\mathbf{x}\|+\|\mathbf{y}\|.$$

When does equality hold? Give a geometric interpretation.

2. Show that for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$,

$$\|\mathbf{x}+\mathbf{y}\|^2 + \|\mathbf{x}-\mathbf{y}\|^2 = 2\|\mathbf{x}\|^2 + 2\|\mathbf{y}\|^2.$$

Give a geometric interpretation.

3. Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and define

$$\mathbf{p} = \left(\frac{\mathbf{x}^T \mathbf{y}}{\mathbf{y}^T \mathbf{y}} \right) \mathbf{y} \quad \text{and} \quad \mathbf{z} = \mathbf{x} - \mathbf{p}.$$

a.) Show (analytically) that $\mathbf{p} \perp \mathbf{z}$. Sketch a picture to illustrate the relationship between $\mathbf{x}, \mathbf{y}, \mathbf{p}$, and $\mathbf{z}$.

b.) If $\|\mathbf{p}\| = 6$ and $\|\mathbf{z}\| = 8$, determine the value of $\|\mathbf{x}\|$.