GP 7

1. $$\|\bar{x}+\bar{y}\|^2 = \langle \bar{x}+\bar{y}, \bar{x}+\bar{y}\rangle$$
$$= \langle \bar{x},\bar{x}\rangle + 2\langle \bar{x},\bar{y}\rangle + \langle \bar{y},\bar{y}\rangle$$
$$= \|\bar{x}\|^2 + 2\langle \bar{x},\bar{y}\rangle + \|\bar{y}\|^2$$
$$\leq \|\bar{x}\|^2 + 2|\langle \bar{x},\bar{y}\rangle| + \|\bar{y}\|^2$$
$$\leq \|\bar{x}\|^2 + 2\|\bar{x}\|\|\bar{y}\| + \|\bar{y}\|^2 \quad \text{by CSB} \neq$$
$$= (\|\bar{x}\| + \|\bar{y}\|)^2$$

Thus, $\|\bar{x}+\bar{y}\| \leq \|\bar{x}\| + \|\bar{y}\|$.

Equality holds only if $\langle \bar{x},\bar{y}\rangle = \|\bar{x}\|\|\bar{y}\|$, whence $\bar{x}$ and $\bar{y}$ would be linearly dependent.

"Triangle Inequality"

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2. "Parallelogram Law"

$\|\bar{x}+\bar{y}\|^2 = \|\bar{x}\|^2 + 2\langle \bar{x},\bar{y}\rangle + \|\bar{y}\|^2$ from above.

$\|\bar{x}-\bar{y}\|^2 = \|\bar{x}\|^2 - 2\langle \bar{x},\bar{y}\rangle + \|\bar{y}\|^2$

Adding these together, the inner products cancel and

$$\|\bar{x}+\bar{y}\|^2 + \|\bar{x}-\bar{y}\|^2 = 2\|\bar{x}\|^2 + 2\|\bar{y}\|^2$$

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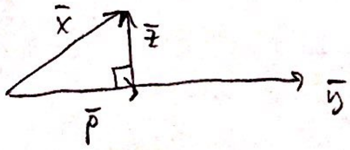
3. a.) $$\langle \bar{p},\bar{z}\rangle = \left\langle \frac{\bar{x}^T\bar{y}}{\bar{y}^T\bar{y}}\bar{y},\ \bar{x} - \frac{\bar{x}^T\bar{y}}{\bar{y}^T\bar{y}}\bar{y} \right\rangle = \left(\frac{1}{\bar{y}^T\bar{y}}\right)^2 \langle (\bar{x}^T\bar{y})\bar{y},\ (\bar{y}^T\bar{y})\bar{x} - (\bar{x}^T\bar{y})\bar{y}\rangle$$
$$= \left(\frac{1}{\bar{y}^T\bar{y}}\right)^2 \left[(\bar{x}^T\bar{y})(\bar{y}^T\bar{y})\langle \bar{y},\bar{x}\rangle - (\bar{x}^T\bar{y})^2\langle \bar{y},\bar{y}\rangle\right]$$

but $\langle \bar{a},\bar{b}\rangle = \bar{a}^T\bar{b}$ for this problem, so we have:

$$\langle \bar{p},\bar{z}\rangle = \left(\frac{1}{\bar{y}^T\bar{y}}\right)^2 \left[\underbrace{(\bar{x}^T\bar{y})(\bar{y}^T\bar{y})(\bar{y}^T\bar{x}) - (\bar{x}^T\bar{y})^2(\bar{y}^T\bar{y})}_{=0.}\right]$$

whence $\bar{p} \perp \bar{z}$.



b.) $\|\bar{p}\|^2 + \|\bar{z}\|^2 = \|\bar{x}\|^2 \Rightarrow \|\bar{x}\| = \sqrt{6^2+8^2} = \sqrt{100} = 10.$