Name:_____

M511: Linear Algebra (Spring 2018)
Instructor: Justin Ryan
Good Problems 8: Sections 5.2–5.3

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Determine bases for each of the four fundamental subspaces associated to the matrix

$$A = \begin{pmatrix} 4 & -2 \\ 1 & 3 \\ 2 & 1 \\ 3 & 4 \end{pmatrix}.$$

2. Let $A \in \mathbb{R}^{m \times n}$. Show that

1. if $\mathbf{x} \in \text{Null}(A^T A)$, then $A\mathbf{x}$ is in both $\text{Col}(A)$ and $\text{Null}(A^T)$.
2. $\text{Null}(A^T A) = \text{Null}(A)$.
3. A and $A^T A$ have the same rank.
4. if A has linearly independent columns, then $A^T A$ is nonsingular.

3. Find the best least squares fit by a linear function to the data.

x	-1	0	1	2
y	0	1	3	9

Plot your linear functions as well as the data points on the same set of axes.

4. Find the equation of the circle that gives the best least squares circle fit to the points $(-1,-2)$, $(0,2.4)$, $(1.1,-4)$, and $(2.4,-1.6)$.