Name:________________________________

M511: Linear Algebra (Spring 2018)

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Good Problems 11: Sections 5.6 and 5.7

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Use the Gram-Schmidt process to find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\mathbf{x}_1 = (4, 2, 2, 1)^T$, $\mathbf{x}_2 = (2, 0, 0, 2)^T$, and $\mathbf{x}_3 = (1, 1, -1, 1)^T$.

2. Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}.$$

a) Use the Gram-Schmidt process to find an orthonormal basis for Col(A).

b) Factor A into a product QR where Q has orthonormal columns and R is upper triangular.

c) Use the factorization in part b to solve the least squares problem $A\mathbf{x} = \mathbf{b}$.

3. Find an orthonormal basis $\{P_0, \ldots, P_4\}$ for $\mathbb{P}_5$ with respect to the inner product

$$\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)\,dx.$$

4. Show that if we choose $P_n(1) = 1$ and define the recursion relation

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

then the polynomials P_n are orthonormal with respect to the inner product

$$\langle p, q\rangle = \int_{-1}^{1} p(x)q(x)dx.$$

These polynomials are known as the *Legendre polynomials.*