Name:________________________________
M511: Linear Algebra
Summer 2018
Midterm Exam: Chapters 1–3 (part I)

WICHITA STATE UNIVERSITY

Instructions. *Complete all problems below, showing enough work. Read carefully and follow all instructions. You may not use any notes or electronic devices. All you need is a pencil and your brain.*

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1. True/False [20 points] *Neatly write* **T** *on the line if the statement is always true, and* **F** *otherwise [1 point each]. In the space provided below the statement, give sufficient explanation of your answer [3 point each].*

_______**1.a.** Every homogeneous linear system is consistent.

_______**1.b.** Let $A \in \mathbb{R}^{n\times n}$. If A is nonsingular and $A = A^{-1}$, then either $A = I$ or $A = -I$.

_______**1.c.** Let $A \in R^{n\times n}$ and $k \in \mathbb{R}$. Then $\det(k \cdot A) = k \cdot \det(A)$.

_______**1.d.** If $\{\mathbf{x}_1, \ldots, \mathbf{x}_k\}$ are vectors in a vector space V and $\text{span}\,\{\mathbf{x}_1, \ldots, \mathbf{x}_k\} = \text{span}\,\{\mathbf{x}_1, \ldots, \mathbf{x}_{k-1}\}$, then $\{\mathbf{x}_1, \ldots, \mathbf{x}_k\}$ are linearly dependent.

_______**1.e.** Let $A \in \mathbb{R}^{m\times n}$. Then $\dim(\text{Null}(A)) = \dim(\text{Null}(A^T))$.

2. The matrix A is nonsingular. Compute its inverse *without* using determinants or cofactors.

$$A = \begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix}$$

3. Find a basis for the null space of the matrix.

$$A = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 2 & -2 & -1 & -2 \\ -1 & 3 & 2 & 1 \end{pmatrix}$$

4. Consider the matrix $A \in \mathbb{R}^{3\times 3}$:

$$A = \begin{pmatrix} 4 & 2 & -1 \\ -2 & 0 & 0 \\ 8 & -2 & 1 \end{pmatrix}$$

a.) Find the *LU* factorization of *A*.

b.) Use your answer to part *a.*) to compute the determinant of *A*.

5. Solve the linear system of equations using Cramer's Rule. Give your answers as reduced fractions. You must use Cramer's Rule to receive credit.

$$\begin{cases} 3x_1 + 5x_2 &= -2 \\ 2x_1 + 4x_2 &= 1 \end{cases}$$

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6. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}.$$

Compute the determinant of A.

7. Find the values of λ for which the matrix $A \in \mathbb{R}^{3\times 3}$ is singular.

$$A = \begin{pmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{pmatrix}$$

8. Solve the linear system of equations, if possible, using your favorite method.

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 - x_2 + x_3 = 3 \\ -x_1 + 2x_2 + 3x_3 = 7 \end{cases}$$

9. Consider the subset of $C^\infty(\mathbb{R})$, $S = \text{span}\left\{e^t, e^{-t}\right\}$.

a.) What is the dimension of S? Justify your answer.

b.) Recall that $\cosh(t) = \frac{1}{2}\left(e^t + e^{-t}\right)$ and $\sinh(t) = \frac{1}{2}\left(e^t - e^{-t}\right)$. Find the transition matrix from $\{e^t, e^{-t}\}$ to $\{\cosh(t), \sinh(t)\}$.

10. Consider the following vectors in $\mathbb{R}^2$,

$$\mathbf{u}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \text{and} \quad \mathbf{v}_2 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$$

Write $\mathbf{x} = 4\mathbf{v}_1 - 2\mathbf{v}_2$ as a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$.