Name:________________________________
M511: Linear Algebra (Summer 2018)
Good Problems 2: Chapter 2

WICHITA STATE UNIVERSITY

Instructions *Complete all problems on this paper, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Let $A, B \in \mathbb{R}^{3\times 3}$ with $\det(A) = 4$ and $\det(B) = 6$, and let E be an elementary matrix of type I. Determine the value of each of the following:

a.) $\det\left(\frac{1}{2}A\right)$

b.) $\det(B^{-1}A^T)$

c.) $\det(EA^2)$

2. If $A \in \mathbb{R}^{n\times n}$ is nonsingular, show that $A^T A$ is nonsingular and $\det(A^T A) > 0$.

3. Let $A \in \mathbb{R}^{n\times n}$ and let λ be a scalar. Show that $\det(A - \lambda I) = 0$ if and only if $A\mathbf{x} = \lambda\mathbf{x}$ for some $\mathbf{x} \neq \mathbf{0}$.

4. Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ with $\mathbf{x} \neq \mathbf{y}$, and let $A \in \mathbb{R}^{n\times n}$. Show that if $A\mathbf{x} = A\mathbf{y}$, then $\det(A) = 0$.

5. Let

$$A = \begin{pmatrix} x & 1 & 1 \\ 1 & x & -1 \\ -1 & -1 & x \end{pmatrix}.$$

a.) Compute all minors and cofactors of A.

b.) Compute $\det(A)$. (Your answer should be a function of x.)

c.) For what values of x will the matrix be singular?

6. Let

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}.$$

a.) Compute the LU factorization of A.

b.) Use the LU factorization to determine the value of $\det(A)$.